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# A MULTI-OBJECTIVE MODEL-PREDICTIVE CONTROL APPROACH DEALING WITH CONGESTION AND EMISSIONS IN URBAN TRAFFIC NETWORKS

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## Abstract

Traffic congestion together with emissions has become a big problem in urban areas. In order to make the best use of the existing road capacity, traffic control systems are widely used. In this paper, we propose a model-predictive controller which uses an integrated flow-emission model as its internal prediction model. The cost function is a weighted combination of the total time spent (TTS) and the total emissions (TE) of the vehicles within the network. We also add the expected time spent and the expected total emission caused by the vehicles that remain in the network at the end of the prediction horizon until they leave the network.

## 1. INTRODUCTION

Traffic congestion has always been a problem in modern urban areas since it is time and energy consuming. Furthermore, traffic emissions are considered as a major risk to people's health and to the environment, since they contain harmful substances including nitrogen oxides ( $\text{NO}_x$ ), hydrocarbon (HC), carbon monoxide (CO), carbon dioxide ( $\text{CO}_2$ ), and particulate matter. To reduce traffic congestion, several solutions have been proposed such as to increase the supply (i.e., the road capacity), to decrease the traffic demand, and to manage the existing capacity. The most feasible approach in the short time consists management of the existing road capacity using appropriate traffic control strategies.

Real-time traffic-responsive control systems are well known as an efficient approach for management of the capacity (Diakaki et al. [2002, 2003]). Among traffic-responsive control approaches, optimization-based control systems and especially model-predictive control (MPC) have proven to be efficient both for freeways and for urban traffic networks (Aboudolas et al. [2010], van den Berg et al. [2007], Bellemans et al. [2006]). In optimization-based control sys-

tems, a cost function is minimized subject to the system dynamics and the structural and physical constraints that are formulated as equality and non-equality equations. In this paper, the focus is on designing an MPC-based controller for an urban traffic network. In MPC the optimization problem is solved along a finite (rolling) horizon, called the prediction horizon, of length  $N_p$  (i.e., the horizon covers  $N_p$  time steps). An MPC-based controller has an internal prediction model that estimates the future states of the system along the prediction horizon. The optimization problem is then solved using the dynamics of the internal model together with the measurements received as feedback at the beginning of the horizon as the initial conditions of the problem. The optimal control signal (i.e., the solution of the optimization problem) is implemented for one time step only, and the optimization problem is solved again at the next time step using the updated measurements as the initial conditions (see Maciejowski [2002] for more details).

A majority of the available literature on urban traffic model predictive control considers the total time spent (TTS) by the vehicles as cost function. While reduction of emissions is also considered as an objective that should be obtained by the control system (Stevanovic et al. [2009], Zegeye et al. [2011], Liao and Machemehl [1996]), only a limited amount of work was carried out in this field.

Some work previously done on MPC for urban traffic networks are as follows; Aboudolas et al. [2010] propose a rolling horizon approach that solves an MPC optimization problem using quadratic programming. The objective of the controller is to minimize and to balance the occupancies of the links so that the risk of over-saturation and spill-back of the queues in the links decreases. A network-wide traffic signal control strategy for congested urban areas is given by Aboudolas et al. [2009], where the modeling is based on the store-and-forward paradigm. Three different control strategies are formulated and discussed: a linear-quadratic optimal control approach, an open-loop quadratic programming control approach, and an open-loop nonlinear optimal control approach. Reduction of traffic emissions is considered by Lin et al. [2013], where an MPC-based controller is designed with the aim to reduce both the congestion and the traffic emissions.

Here, we propose a model-predictive controller that optimizes a multi-objective cost function that includes both the emissions and the congestion. In formulating the objective function we will add two additional terms that account for the expected travel time and the expected emissions of the vehicles that remain in the network at the end of the prediction time.

## 2. INTEGRATED FLOW-EMISSION MODEL FOR URBAN TRAFFIC NETWORKS

In order to design a model-predictive controller, we need a model of the system that predicts the states of the system as well as the emissions, and at the same time provides a balanced trade-off between computational efforts and accuracy of the predictions. The S-model proposed by Lin et al. [2012] is a macroscopic urban flow model. Using the S-model all traffic scenarios including under-saturated, saturated, and over-saturated are covered. The time step of the S-model differs from link to link, i.e., the simulation time step for a link  $(u, d)$  that connects the (upstream) intersection  $u$  and the (downstream) intersection  $d$  is considered to be  $c_d$ , which is equal to the cycle time  $c_d$  of intersection  $d$ . The state variables of the S-model are the number  $n_{u,d}$  of vehicles and the queue lengths  $q_{u,d,o}$  in link  $(u, d)$  that intend to move towards the destination  $o$ . These state variables are updated by:

$$n_{u,d}(k_d + 1) = n_{u,d}(k_d) + (\alpha_{u,d}^{\text{enter}}(k_d) - \alpha_{u,d}^{\text{leave}}(k_d)) c_d \quad (1)$$

$$q_{u,d,o}(k_d + 1) = q_{u,d,o}(k_d) + (\alpha_{u,d,o}^{\text{arriv}}(k_d) - \alpha_{u,d,o}^{\text{leave}}(k_d)) c_d \quad (2)$$

where  $k_d$  is the time step counter,  $\alpha_{u,d}^{\text{enter}}$  and  $\alpha_{u,d}^{\text{leave}}$  are the entering and exiting flow rates according to link  $(u, d)$ ,  $\alpha_{u,d,o}^{\text{arriv}}$  and  $\alpha_{u,d,o}^{\text{leave}}$  are the arriving and the leaving average flow rate of the sub-stream moving towards  $o$ . Note that in the S-model  $n_{u,d}$  and  $q_{u,d,o}$  are approximated using real values.

Different parameters play a role in production of emissions, including the speed and the acceleration of the vehicles. There are some models, such as COPERT (Kouridis et al. [2000]), that consider the effect of the average speed on the emissions. However, the speed alone seems not to be sufficient for accurate estimation of emissions. Therefore, here we use VT-micro (Ahn et al. [2002]), which is a microscopic urban emission model that estimates the emissions  $E_{\theta,i}$  of any type  $\theta \in \{\text{CO}, \text{NO}_x, \text{HC}\}$  produced by the vehicle  $i$  at time step  $k$  considering both the speed and the acceleration of vehicles. The emissions  $E_{\theta,i}$  are given by the following equation using the VT-micro model:

$$E_{\theta,i}(v_i(k), a_i(k)) = \exp(\tilde{v}_i(k) P_{\theta} \tilde{a}_i(k)) \quad (3)$$

with  $P_{\theta}$  a pre-calibrated matrix given by Ahn et al. [2002], and:

$$\tilde{v}_i(k) = [1 \quad v_i(k) \quad v_i^2(k) \quad v_i^3(k)]^{\top}, \quad \tilde{a}_i(k) = [1 \quad a_i(k) \quad a_i^2(k) \quad a_i^3(k)]^{\top}$$

In this paper we use the integrated version of the S-model and the VT-micro model given by Lin et al. [2013].

### 3. MODEL-PREDICTIVE CONTROL

Our proposed approach for real-time signal control of an urban traffic network includes a model-predictive controller. As we explained it before in Section 1, the optimization problem is solved online over the prediction horizon at each control time step. Suppose that the control time interval  $T_{\text{ctrl}}$  is the same for all intersections. Then we have  $T_{\text{ctrl}} = N_d \cdot c_d$  for all  $d \in \mathcal{J}$  where  $\mathcal{J}$  is the set of all intersections in the network, and  $N_d$  is an integer, and thus  $k_d = N_d \cdot k_{\text{ctrl}}$ . Here, we aim to find a balanced trade-off between reduction of the congestion (i.e., reduction of the total time spent (TTS) by the vehicles), and reduction of the total emissions (TE). Therefore, we have a multi-objective optimization problem, for which we define the objective function as a linear weighted combination of different objectives:

$$\begin{aligned} \mathcal{J}(k_{\text{ctrl}}) = & w_1 \frac{\text{TTS}}{\text{TTS}_{\text{nominal}}} + \sum_{\theta \in \Theta} w_{2,\theta} \frac{\text{TE}_{\theta}}{\text{TE}_{\theta,\text{nominal}}} + \\ & w_3 \frac{\text{TTS}_{\text{end-point}}(k_{\text{ctrl}})}{\text{TTS}_{\text{end-point,nominal}}} + \sum_{\theta \in \Theta} w_{4,\theta} \frac{\text{TE}_{\theta,\text{end-point}}(k_{\text{ctrl}})}{\text{TE}_{\theta,\text{end-point,nominal}}} + w_5 \frac{\text{Var}(g)}{\text{Var}_{\text{nominal}}} \end{aligned} \quad (4)$$

where  $\text{Var}(g)$  is the variations in the control signal  $g$  (this term is added to the objective function to suppress possible oscillations of the control signal), and TTS and  $\text{TE}_{\theta}$  denote the total time spent and the total estimated emission of  $\theta \in \Theta = \{\text{CO}, \text{NO}_x, \text{HC}\}$  in the network during the prediction interval, i.e.,:

$$\text{TTS} = \sum_{i=k_{\text{ctrl}}}^{k_{\text{ctrl}}+N_p} \text{TTS}(i), \quad \text{TE}_{\theta} = \sum_{i=k_{\text{ctrl}}}^{k_{\text{ctrl}}+N_p} \text{TE}_{\theta}(i)$$

with  $\text{TTS}(i)$  and  $\text{TE}_{\theta}(i)$  the total time spent and the total emissions during one time step  $[ik_{\text{ctrl}}, (i+1)k_{\text{ctrl}})$ , and  $\text{TTS}_{\text{nominal}}$  and  $\text{TE}_{\theta,\text{nominal}}$  the nominal performances for TTS and  $\text{TE}_{\theta}$ . The 3<sup>rd</sup> and the 4<sup>th</sup> terms in (4) correspond to the vehicles that have entered the network

within the prediction interval  $[k_{\text{ctrl}}c_d, (k_{\text{ctrl}} + N_p)c_d]$  and that are still in the network at the end of the prediction time interval. These two terms consider the expected time spent and the expected emissions resulted by the remaining vehicles until they leave the network. Next we explain how these terms are computed. Suppose that:

- we have a destination independent model,
- traffic situation is fixed after the prediction interval.

For every pair of links  $(x, y), (y, z) \in L$ , with  $L$  the set of all links in the network, we define::

$$\beta_{\text{end-point},x,y,z} = \beta_{\text{ctrl},x,y,z}(k_{\text{ctrl}} + N_p) \quad (5)$$

Now we consider a given link  $(u, d)$ , and for the link we define :

$$n_{\text{end-point},u,d} = n_{\text{ctrl},u,d}(k_{\text{ctrl}} + N_p) \quad (6)$$

If we consider all the possible routes to the end-points of the network that are reachable via  $(u, d)$ , for some networks such as grid-shaped networks vehicles might move within cycles; then the number of the routes will become infinity, and hence the 3<sup>rd</sup> and the 4<sup>th</sup> terms in (4) will grow exponentially. To prevent this situation, we determine a limited number  $K_{u,d}$  of the most likely used routes for the link to the end-points of the network. The aim is to use an existing shortest path algorithm for this, in particular we want to use Yen's  $K$  shortest path routing algorithm (Yen [1971]).

First, the problem should be transformed into a point-to-point problem by connecting all the end-points of the network to a single virtual end-point “ $v$ ” (see Figure 1) so that the problem reduces to finding the  $K_{u,d}$  shortest routes that connect  $d$  (for all  $d \in \mathcal{J}$ ) and  $v$ . A route  $R_j$  between  $d$  and  $v$  is defined as:

$$R_j = \{(d, d_{j,1}), (d_{j,1}, d_{j,2}), \dots, (d_{j,n_j-1}, v)\} \quad (7)$$

with  $n_j$  the number of links in route  $R_j$ .

In our problem, we look for the  $K_{u,d}$  most likely used routes from  $(u, d)$  to  $v$ , i.e., the routes with the largest  $\prod_{(x,y),(y,z) \in (R_j \cup (u,d))} \beta_{\text{end-point},x,y,z}$  for  $j \in \{1, 2, \dots, n_j\}$ , and this is equivalent to finding the largest  $\log \left( \prod_{(x,y),(y,z) \in (R_j \cup (u,d))} \beta_{\text{end-point},x,y,z} \right)$ . Yen's algorithm seeks for the minimized summation of the costs, hence our problem could be reformulated as finding the least  $\sum_{(x,y),(y,z) \in (R_j \cup (u,d))} (-\log \beta_{x,y,z})$ , i.e., we look for the  $K_{u,d}$  shortest routes where the costs  $C(y, z)$  of the links are redefined as (see Figure 1):

$$C(y, z) = -\log \beta_{x,y,z} \quad (8)$$

Note that (8) is a legitimate definition for the cost, as  $0 \leq \beta_{x,y,z} \leq 1$  and hence  $C(y, z) \geq 0$ .

Now suppose that we have found the  $K_{u,d}$  shortest routes from the current link  $(u, d)$  to the end-points of the network. Then we put them all in a set called  $\mathcal{R}_{u,d,K_{u,d}}$ .

Moreover, at  $d$  the summation of the turning rates should be unity. Indeed, by considering only  $K_{u,d}$  routes, we assume that the turning rates towards the other routes are zero. Therefore, we need to define  $\gamma_{u,d,r}$  for  $r \in \{1, 2, \dots, K_{u,d}\}$  where  $\gamma_{u,d,r}$  indicates the percentage of the vehicles that are within link  $(u, d)$  at the end of the prediction interval and that will travel the  $r^{\text{th}}$  route. Then  $\gamma_{u,d,r}$  for  $r \in \{1, 2, \dots, K_{u,d}\}$  is defined as:

$$\gamma_{u,d,r} = \frac{\prod_{(x,y),(y,z) \in (R_r \cup (u,d))} \beta_{\text{end-point},x,y,z}}{\sum_{l=1}^{K_{u,d}} \left( \prod_{(x,y),(y,z) \in (R_l \cup (u,d))} \beta_{\text{end-point},x,y,z} \right)} \quad (9)$$

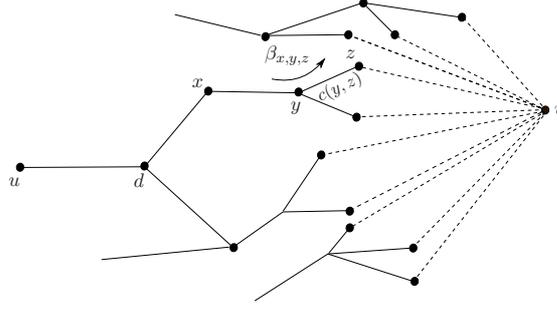


Figure 1: Recast the problem of finding the  $K_{u,d}$  most likely used routes into a point-to-point  $K$  shortest path problem

Now that the least costly routes are determined, we can compute  $TTS_{\text{end-point}}$  and  $TE_{\theta,\text{end-point}}$  in (4) as follows:

$$TTS_{\text{end-point}}(k_{\text{ctrl}}) = \sum_{(u,d) \in L} \left( n_{\text{end-point},u,d} \sum_{r=1}^{K_{u,d}} \gamma_{u,d,r} TTS_{u,d,r} \right) \quad (10)$$

$$TE_{\theta,\text{end-point}}(k_{\text{ctrl}}) = \sum_{(u,d) \in L} \left( n_{\text{end-point},u,d} \sum_{r=1}^{K_{u,d}} \left( \gamma_{u,d,r} \sum_{\theta \in \Theta} TE_{\theta,u,d,r} \right) \right) \quad (11)$$

where  $TTS_{u,d,r}$  and  $TE_{\theta,u,d,r}$  are the total time spent and the total emissions of  $\theta$  along route  $r$ .

#### 4. FUTURE WORK

For future work, the theory proposed in this paper will be applied to an urban traffic model to evaluate the efficiency of the proposed controller. Moreover, smoothing methods will be applied to the flow S-model so that the efficient available optimization algorithms could be applied for the smooth formulation.

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#### REFERENCES

- K. Aboudolas, M. Papageorgiou, and E. Kosmatopoulos. Store-and-forward based methods for the signal control problem in large-scale congested urban road networks. *Transportation Research Part C*, 17(2):163–174, 2009.
- K. Aboudolas, M. Papageorgiou, A. Kouvelas, and E. Kosmatopoulos. A rolling-horizon quadratic-programming approach to the signal control problem in large-scale congested urban road networks. *Transportation Research Part C*, 18(5):680–694, 2010.

- K. Ahn, H. Rakha, A. Trani, and M. van Aerde. Estimating vehicle fuel consumption and emissions based on instantaneous speed and acceleration levels. *Journal of Transportation Engineering*, 128(2):182–190, 2002.
- T. Bellemans, B. De Schutter, and B. De Moor. Model predictive control for ramp metering of motorway traffic: A case study. *Control Engineering Practice*, 14(7):757–767, 2006.
- C. Diakaki, M. Papageorgiou, and K. Aboudolas. A multi-variable regulator approach to traffic-responsive network-wide signal control. *Control Engineering Practice*, 10(2):183–195, 2002.
- C. Diakaki, V. Dinopoulou, K. Aboudolas, M. Papageorgiou, E. Ben-Shabat, E. Seider, and A. Leibov. Extensions and new applications of the traffic-responsive urban control strategy: Coordinated signal control for urban networks. *Transportation Research Record*, 1856:202–211, 2003.
- C. Kouridis, L. Ntziachristos, and Z. Samaras. COPERT III Computer programme to calculate emissions from road transport: Methodology and emission factors (version 2.1). Technical Report 49, European Environment Agency, Copenhagen, Denmark, 2000.
- T. Y. Liao and R. Machemehl. Optimal traffic signal strategy for fuel consumption and emissions control at signalised intersections. In *24th European Transport Forum*, England, Sept. 1996.
- S. Lin, B. De Schutter, Y. Xi, and H. Hellendoorn. Efficient network-wide model-based predictive control for urban traffic networks. *Transportation Research Part C*, 24:122–140, Oct. 2012.
- S. Lin, B. De Schutter, Y. Xi, and H. Hellendoorn. Integrated urban traffic control for the reduction of travel delays and emissions. *IEEE Transactions on Intelligent Transportation Systems*, 1(4):1609–1619, 2013.
- J. Maciejowski. *Predictive Control with Constraints*. Prentice Hall, London, UK, 2002.
- A. Stevanovic, J. Stevanovic, K. Zhang, and S. Batterman. Optimizing traffic control to reduce fuel consumption and vehicular emissions. *Transportation Research Record*, 2128:105–113, 2009.
- M. van den Berg, A. Hegyi, B. De Schutter, and J. Hellendoorn. Integrated traffic control for mixed urban and freeway networks: A model predictive control approach. *European Journal of Transport and Infrastructure Research*, 7(3):223–250, 2007.
- J. Yen. Finding the K shortest loopless path in a network. *Management Science*, 17(11): 712–716, 1971.
- S. Zegeye, B. De Schutter, J. Hellendoorn, and E. Breunese. Reduction of area-wide emissions using an efficient model-based traffic control strategy. In *Proceedings of the 2011 IEEE Forum on Integrated and Sustainable Transportation Systems (FISTS 2011)*, pages 307–312, Vienna, Austria, June–July 2011.